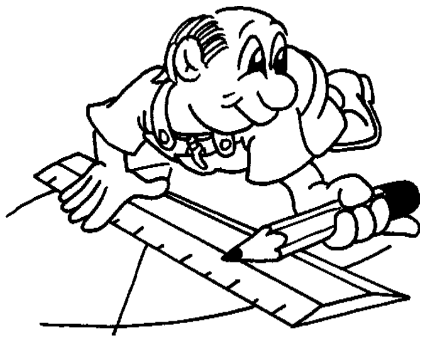
The Mathematics of Circles

Mathematics activities designed to extend and challenge 11 to 13 year olds.



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FOREWORD

This book is divided into three sections. The first section on circle geometry contains exercises which are of an explanatory nature, and involve some compass and protractor work. The geometry concepts are explored informally, and students are asked to write down their findings.

The second, on circle arithmetic, provides extension exercises for children who have been taught to calculate circumference and area of a circle.

The final section contains a variety of activities, all loosely based on circles. Some are compass constructions, which require accurate use of compass and protractor, as well as a steady hand! There are also some number and logic puzzles.

There is something for everyone in this book. Enjoy it!

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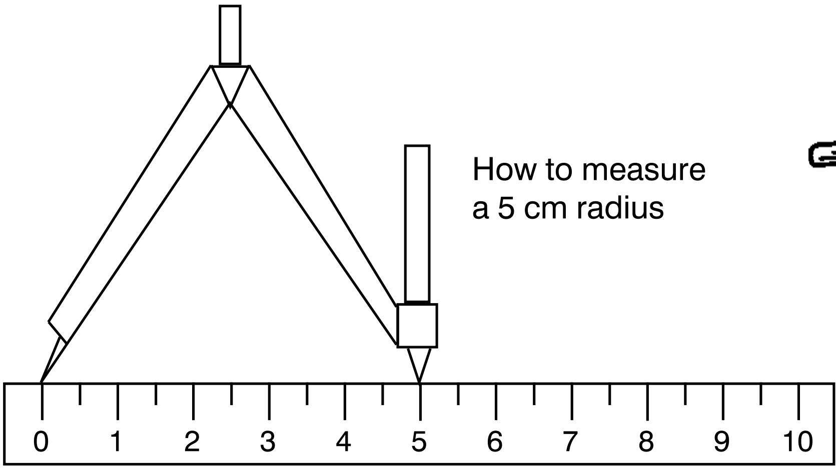
Name ....

Drawing A Circle

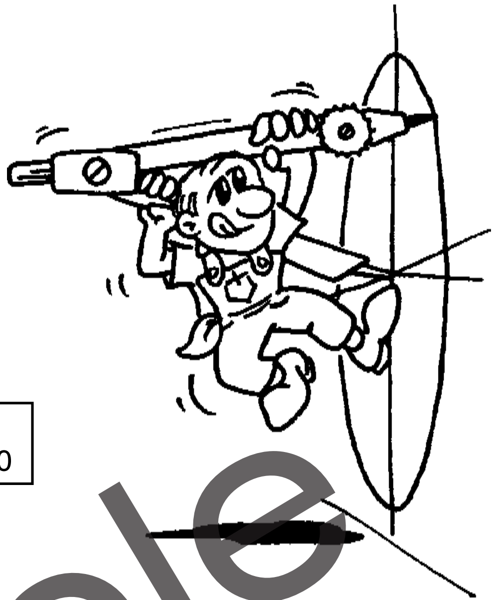
EQUIPMENT NEEDED

Compass, ruler.

You need a firm compass which will not slip, and which has a sharp pencil and a good point.



How to measure
a 5 cm radius



❑ 1. Draw a circle with a 2 cm radius.
Using the same centre, draw
another with a 3 cm radius.
These circles are ***concentric***.

❑ 2. Draw a circle with a 3 cm radius below.
Now draw another 3 cm circle with its centre on the circumference of the first circle.

What do you notice about the two centres? ....

....

How far apart are the centres? ....

❑ Draw a line through both centres, so that it touches each circle twice.

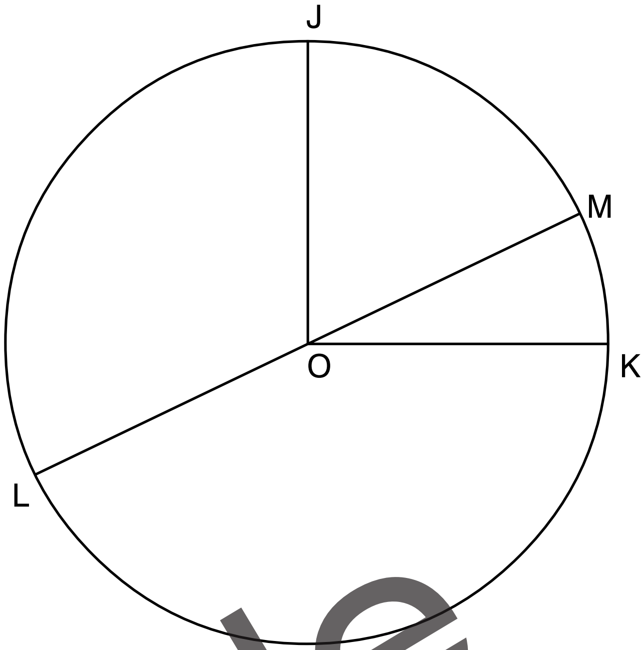
How long is the line between the furthest points on the circles? ....

Name ....

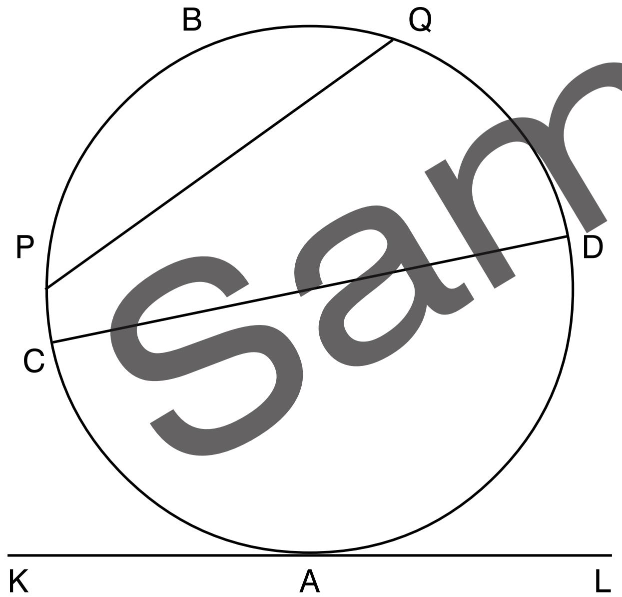
Parts Of A Circle

EQUIPMENT NEEDED
Ruler, pencil, eraser.

This circle has its ***centre*** at O.
$\overline{JO}$ is a ***radius***.
$\overline{KO}$ is another radius.
$\overline{LM}$ is a ***diameter***.
$\overline{LM}$ also contains two ***radii***, $\overline{LO}$ and $\overline{OM}$



Draw a radius $\overline{PO}$.
Draw a diameter $\overline{QR}$.
Use a different coloured pencil
to trace over the radius $\overline{OQ}$.



$\overline{PQ}$ is a ***chord*** of this circle.
Arc PAQ is the ***major*** (longer) arc.
Arc PBQ is the ***minor*** (shorter) arc.
Arc CBD is a ***semi circle***.

Line $\overline{KAL}$ is called a ***tangent***. It touches the circle in exactly one place, at A.

Draw tangents through B, P, Q, C, D.

What do you notice about the tangents through C and D?

They are ....

Name ....

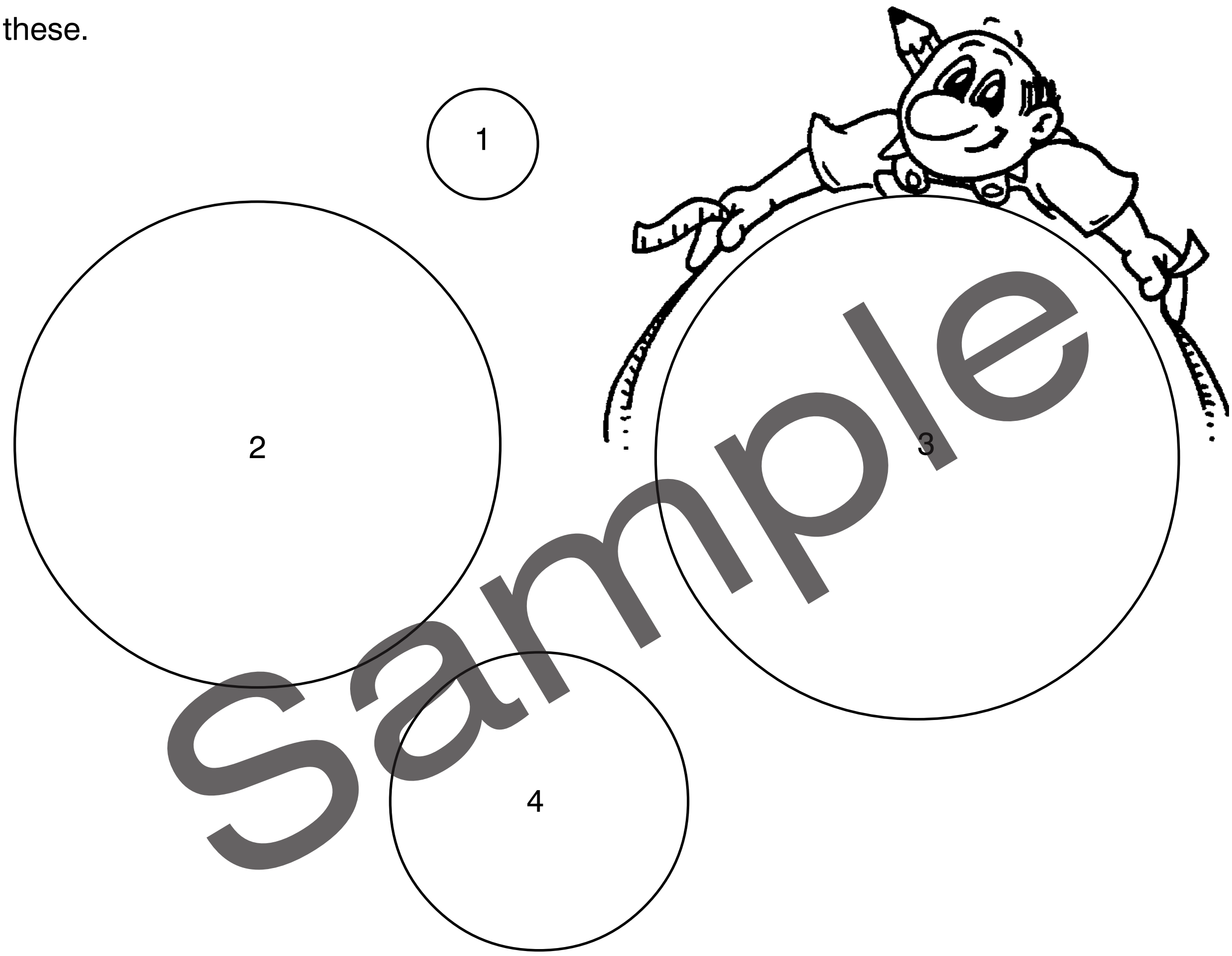
How Long Is A Circle?

EQUIPMENT NEEDED
Thread, string etc., cylindrical objects for measurement, ruler or tape measure.

Here are three ways of estimating the circumference of a circle, if you only need to know its approximate length.

1. Take a piece of string, chain or thread (or hair), and carefully lay it round the circle. Then measure the length of thread you wrapped round the circle.

Try these.



2. Find a cylinder of about the same size as your circle, and measure around that. This could be a can or tube or bottle.

3. Measure the diameter of the circle. When you multiply this by 3, you will be just under the actual circumference.

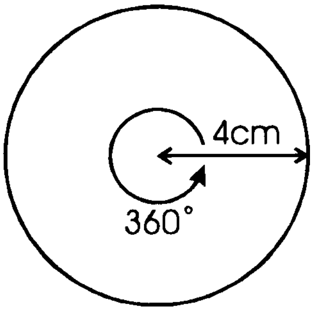
❑ Try it with the circles above.

circle 1 = ... circle 2 = ...

circle 3 = ... circle 4 = ...

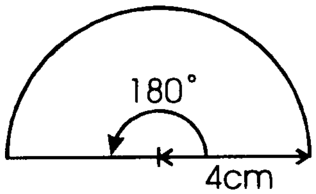
Parts Of The Circumference

EQUIPMENT NEEDED
Calculator - optional.



This circle has a radius of 4 cm.
The circumference is

$C = 2\pi r$
$= 2 \times \pi \times 4$ use $\pi = 3{\cdot}14$
$= 25{\cdot}12$ cm.



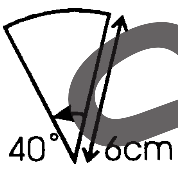
Here is half the circle. It is called an arc of the circle.
The angle at the centre of a whole circle is 360°, so the angle at the centre of half a circle is 180°.
The circumference from P to Q is half of the full circle.

Arc PQ $= \frac{180}{360} \times 25{\cdot}12$

$= 12{\cdot}56$ cm.

> If the radius of a circle is r,
> If the angle at the centre is A°,
> Then the length of the arc is   Arc $= \frac{A}{360} \times 2\pi r$

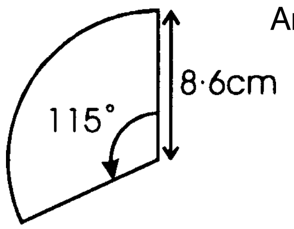
❑ Try these:

1. 

Arc $= \frac{40}{360} \times 2\pi r$

$= \frac{40}{360} \times 2 \times 3{\cdot}14 \times 6$

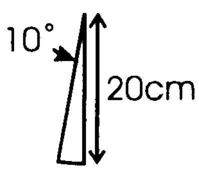
= .... cm

2. 

Arc = ...

= ....

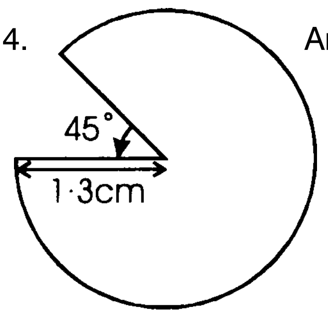
= ....

3. 

Arc = ...

= ....

= ....

4. 

Arc = ...

= ....

= ....

❑ On the back of this page, draw and calculate these:

5. radius 12·6 m, angle 63°
6. radius 25 cm, angle 155·6°
7. radius 38 m, angle 46°
8. radius 80 cm, angle 212°

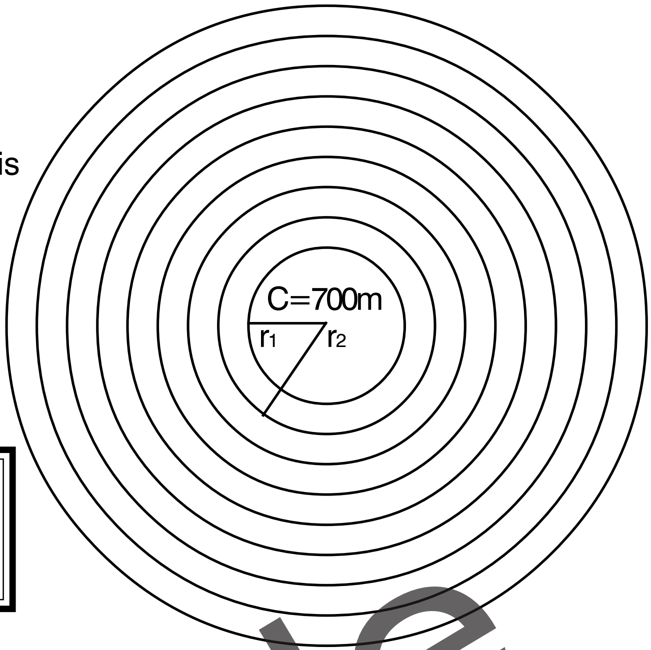
Name ....

On The Right Track

EQUIPMENT NEEDED
Calculator - optional.

The Roundsville Athletic Club has a circular running track.

The track is designed so that the inside lane is exactly 700 metres long on the shorter side. There are eight lanes marked.



❑ a) Calculate the radius r_1, of the inside (shorter side) of the inside lane to nearest 1 decimal place.

We know that $C = 2 \times \pi \times r_1$

and that $C = 700$ m

$\therefore 700 = 2 \times \pi \times r_1$

$r_1 =$

Each lane is 1 metre wide.

❑ b) What is the radius r_2 of the inside of the second lane? ....

❑ c) Calculate the distance c_2 that the runner in lane 2 will run, if he stays on the inside of his lane.

$c_2 =$ ....

$=$ ....

❑ d) Fill in the table. (Do your working on another sheet of paper)

Lane	Radius	Track Length
1		
2		
3		
4		
5		
6		
7		
8		

❑ e) What is the difference (to the nearest metre) in the lengths of any two neighbouring lanes?

❑ f) What is the length of the outside fence around the perimeter of the running tracks?

❑ g) On another sheet of paper, draw a diagram of the track, showing how the starting places for the runners must be staggered, so that all runners run the same distance to a straight finishing line.

Name ....

Twenty Fours

EQUIPMENT NEEDED
Calculator - optional.

Here is a number challenge for you.

8	4
5	7

4	7
2	2

The object of this game is to make 24 by using all of the numbers in the circle, with a single operator ($+ - \times \div$) between each pair.
Like this:

$8 + 4 + 5 + 7 = 24$

This one is a little harder!
(You may need to use brackets here)

$(4 \times 7) - (2 + 2) = 24$

or $(4 \times 7) - 2 - 2 = 24$

You try these: (There may be more than one correct way to do them)

1.

8	6
2	2

2.

8	7
8	1

3.

8	8
1	4

4.

6	4
7	7

5.

9	9
3	7

6.

1	2
6	8

7.

3	6
3	2

8.

1	8
5	8

9.

5	5
2	9

10.

8	4
2	2

11.

3	9
5	3

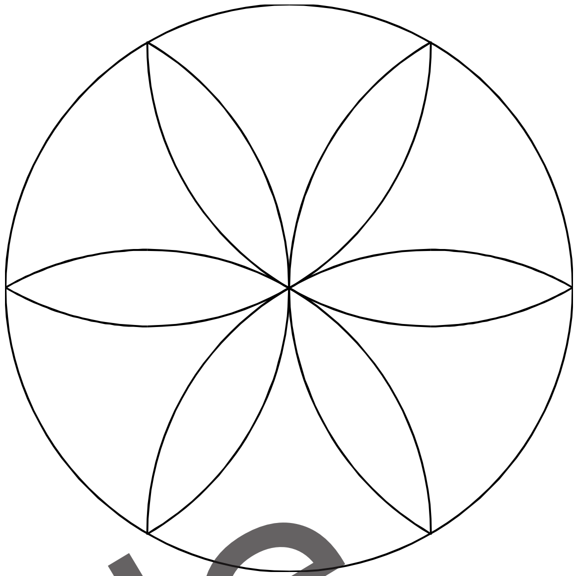
12.

4	4
4	2

Name ....

Compass Flower

EQUIPMENT NEEDED
Compass.



Here's how you draw one of these:

1. Draw a circle with radius 6 cm in the space below.
2. Keeping your compass at 6 cm radius, put the point on the circle and draw an arc from one side of the circle to the other.
3. Now shift the compass point to where the arc meets the circle. Repeat step 2.
4. Repeat these steps until you return to your starting point (6 arcs).

THERE'S YOUR FLOWER.

What to do next:

❑ You could:
Cut out the flower.
Colour it in.
Make more petals, evenly spaced between the ones you already have.
Make some flowers out of card or foil for decorations.